UNSPLIT-FIELD FORMULATIONS FOR GENERALISED MATERIAL INDEPENDENT PML ABSORBERS

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ABSTRACT

To absorb EM waves propagating in arbitrary anisotropic media consisting of both permittivity and permeability tensors, a generalized material independent PML (GMIPML) absorber based on split-field formulations was recently developed by the author. In this paper, to further enhance the efficiency of the GMIPML, unsplit-field formulations for this absorber are proposed. Validation of the GMIPML based on the unsplit-field formulations is carried out through numerical examples.

INTRODUCTION

The perfectly matched layer (PML) absorbing boundary condition developed by Berenger [1] has been widely accepted as one of the best ABCs for the truncation of FDTD lattices. Very recently, the material independent PML (MIPML) absorber [2] (used for arbitrary anisotropic dielectric media) and the generalized material independent PML (GMIPML) absorber [3] (used for arbitrary anisotropic dielectric and magnetic media), have been proposed. The most important advantage of the absorbers [2-3] over the previous PML absorbers [4-8] is that they have the property of material independence, due to the introduction of the material independent quantities (i.e., electric displacement **D** and flux density **B**) into the FDTD model. Consequently, Berenger's PML can be simply and effectively extended to arbitrary anisotropic media with the help of the GMIPML absorber. Nevertheless, the implementation of the approaches proposed in [2-3] requires splitting of the **D** and **B** fields, which certainly affects their efficiency. In this paper, to further enhance the efficiency of the GMIPML absorber, unsplit-field formulations of the GMIPML absorber used for 3-D arbitrary anisotropic materials consisting of permittivity and permeability tensors are derived and validated through numerical examples. It should be noticed that although unsplit-field formulations (based on diagonal anisotropic PMLs) for PML absorbers were developed [5-8], they cannot be applied to *arbitrary* anisotropic materials.

THEORY

As indicated in [2, 3, 9], once **D** and **B** are introduced into the FDTD model, the GMIPML absorbers containing conductivities σ^D and σ^B can be constructed, and the matching condition for the GMIPML is: $\sigma_p^D = \sigma_p^B$ (where p = x, y, or z). If waves propagate in 3-D arbitrary anisotropic media consisting of relative permittivity tensor $[\varepsilon]$ and permeability tensor $[\mu]$, then following the works [7-8], Maxwell's equations for the face-GMIPML absorber in z direction are:

$$\nabla\times\mathbf{E} = -j\omega[A^z]\mathbf{B} \quad (1.1)$$

$$\nabla\times\mathbf{H} = j\omega\,[A^z]\mathbf{D} \quad (1.2)$$

$$\mathbf{E} = (1/\varepsilon_0)[\varepsilon]^{-1}\mathbf{D} \quad (1.3)$$

$$\mathbf{H} = (1/\mu_0)[\mu]^{-1}\mathbf{B} \quad (1.4)$$

where $[A^z]$ = diag{s_z, s_z $1/s_z$}, and $s_z = 1 + \sigma_z^D/j\omega$. Furthermore, Eq. (1) can be written in another forms, e.g., from Eq. (1.2) one has:

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$$j\omega\left(1+\frac{\sigma_z^D}{j\omega}\right)D_x=\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z} \qquad (2.1)$$

$$j\omega\left(1+\frac{\sigma_z^D}{j\omega}\right)D_y=\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x} \qquad (2.2)$$

$$j\omega D_z\left(1+\frac{\sigma_z^D}{j\omega}\right)^{-1}=\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y} \qquad (2.3)$$

From Eq. (2) one sees that the D_x and D_y field components can be computed via the standard explicit FDTD update formulation, e. g.,

$$D_x^{n+1}(i,j,k)=\frac{2-\sigma_z^D\Delta t}{2+\sigma_z^D\Delta t}D_x^n(i,j,k)+$$
$$\frac{2\Delta t}{2+\sigma_z^D\Delta t}\left[\frac{H_z^{n+0.5}(i,j+1,k)-H_z^{n+0.5}(i,j,k)}{\Delta y}\right.$$
$$\left.-\frac{H_y^{n+0.5}(i,j,k+1)-H_y^{n+0.5}(i,j,k)}{\Delta z}\right] \qquad (3)$$

Although D_z cannot be updated with the above manner, it can be updated with a updating procedure similar to the two-step approach [8]. In particular, if a normalized electric displacement $\overline{D}_z$ is introduced as: $\overline{D}_z=D_z/\left[1+(\sigma_z^D/j\omega)\right]$, then Eq. (2.3) can be first updated with the standard FDTD formulation:

$$\overline{D}_z^{n+1}(i,j,k)=\overline{D}_z^n(i,j,k)+$$
$$\Delta t\left[\frac{H_y^{n+0.5}(i+1,j,k)-H_y^{n+0.5}(i,j,k)}{\Delta x}\right.$$
$$\left.-\frac{H_x^{n+0.5}(i,j+1,k)-H_x^{n+0.5}(i,j,k)}{\Delta y}\right] \qquad (4)$$

Once $\overline{D}_z$ is calculated, D_z can be updated from $\overline{D}_z$. Especially, the updating equation for D_z is:

$$D_z^{n+1}=D_z^n+\left(\overline{D}_z^{n+1}-\overline{D}_z^n\right)+\frac{\sigma_z^D\Delta t}{2}\left(\overline{D}_z^{n+1}+\overline{D}_z^n\right) \qquad (5)$$

Eqs. (3-5) are the updating equations for the face-GMIPML in z direction. Similar equations can be derived for the face-GMIPML in x and y directions. Furthermore, updating equations for the edge and corner regions of the GMIPML can be obtained with a similar manner as those used in [7-8]. For instance, in the corner region of the GMIPML, Eq. (1) is still valid when $[A^z]$ is replaced by $[A^{xyz}]$, where $[A^{xyz}]=[A^x][A^y][A^z]=\mathrm{diag}\{(s_ys_z)/s_x,\ (s_xs_z)/s_y,\ (s_xs_y)/s_z\}$. From the above discussion, one can easily see that once the conductivities σ^E and σ^H are replaced by σ^D and σ^B the GMIPML can be constructed in a similar manner as those used in 3-D PML for isotropic or diagonal anisotropic media [7-8]. However, in the GMIPML absorber **E** (or **H**) are updated from the relation $\mathbf{E}=(1/\varepsilon_0)[\varepsilon]^{-1}\mathbf{D}$ (or $\mathbf{H}=(1/\mu_0)[\mu]^{-1}\mathbf{B}$) with an averaging approximation procedure on the components of the **D** (or **B**) field in space. For instance, the updating equation for E_x in the GMIPML absorber (also inside the anisotropic material) is:

$$E_x^n(i,j,k)=\frac{(\varepsilon_{yy}\varepsilon_{zz}-\varepsilon_{yz}\varepsilon_{zy})}{\Lambda}D_x^n(i,j,k)+$$
$$\frac{(\varepsilon_{xz}\varepsilon_{zy}-\varepsilon_{xy}\varepsilon_{zz})}{4\Lambda}\left[D_y^n(i,j,k)+D_y^n(i+1,j,k)\right.$$
$$\left.+D_y^n(i+1,j+1,k)+D_y^n(i,j+1,k)\right]+$$
$$\frac{(\varepsilon_{xy}\varepsilon_{yz}-\varepsilon_{xz}\varepsilon_{yy})}{4\Lambda}\left[D_z^n(i,j,k)+D_z^n(i+1,j,k)\right.$$
$$\left.+D_z^n(i+1,j,k+1)+D_z^n(i,j,k+1)\right] \qquad (6)$$

where $\Lambda=\varepsilon_0(\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz}+\varepsilon_{xy}\varepsilon_{yz}\varepsilon_{zx}+\varepsilon_{xz}\varepsilon_{yx}\varepsilon_{zy}-\varepsilon_{xz}\varepsilon_{yy}\varepsilon_{zx}-\varepsilon_{xy}\varepsilon_{yx}\varepsilon_{zz}-\varepsilon_{xx}\varepsilon_{yz}\varepsilon_{zy})$. The updating equations for the rest components of the **E** field can be derived analogically. Eq. (6) indicates that **E** can be calculated from **D**. This also means that inside the GMIPML absorber **E** can be *simultaneously* absorbed while **D** is *directly* absorbed. The above statement applies to **H** and **B** too.

NUMERICAL VALIDATION

To validate the GMIPML absorber based on the unsplit-field formulation, waves propagating in a 3-D uniaxial anisotropic (loss-free) medium consisting of both dielectric ($\varepsilon_1 = 2$ and $\varepsilon_2 = 2.2$) and magnetic ($\mu_1 = 2$ and $\mu_2 = 2.2$) materials are studied. In particular, if θ is the angle between optical axis (within the xz-plane) and x direction, then the non-zero elements of the permittivity tensor $[\varepsilon]$ are:

$$\varepsilon_{xx} = \varepsilon_1 \cos^2\theta + \varepsilon_2 \sin^2\theta \tag{7.1}$$

$$\varepsilon_{yy} = \varepsilon_1 \tag{7.2}$$

$$\varepsilon_{zz} = \varepsilon_1 \sin^2\theta + \varepsilon_2 \cos^2\theta \tag{7.3}$$

$$\varepsilon_{xz} = \varepsilon_{zx} = (\varepsilon_2 - \varepsilon_1)\sin\theta\cos\theta \tag{7.4}$$

whereas the non-zero elements of the permeability tensor $[\mu]$ are fixed as: $\mu_{xx} = 2.05$, $\mu_{yy} = 2.15$, $\mu_{zz} = 2.0$, and $\mu_{xy} = \mu_{yx} = 0.0866$. In the simulations, the following parameters are used: total computational domain (including the GMIPML) is $40 \times 40 \times 41$, space steps are $\Delta x = \Delta y = \Delta z = 6$ mm, time step is $\Delta t = 12.5$ ps, number of the GMIPML cells (in all the x, y, and z directions) is $N = 10$, and normal theoretical reflection R(0) is 0.01%. With such values of N and R(0) the optimal value for the power of the conductivity $\sigma^D(\rho)$ profile is $n = 4$ [3]. The system is excited by D_z with a smooth compact pulse located at the center point of the domain. The FDTD reference solution is computed with a large computational domain, and all results are recorded at 120 time steps.

To evaluate the performance of the GMIPML absorber based on the unsplit-field formulation, Fig. 1 shows the normalized (with respect to the maximum value of the reference solution within the region $11\Delta x \leq x \leq 31\Delta x$) local reflections (for the cases $\theta = \pi/6$, $\pi/4$, and $\pi/3$, respectively), caused by the GMIPML, for the E_z field component along a special line (x, 11, 20.5) (i.e., the interface. between the anisotropic medium and the GMIPML absorber). The results in Fig. 1 indicate that the proposed GMIPML performs quite well (better than -60dB) for all the different anisotropic cases.

CONCLUSIONS

To further improve the efficiency of the GMIPML absorber, formulations without splitting the **D** and **B** fields are proposed. In contrast with the previous 3-D PML absorbers based on the unsplit-field formulations used for isotropic and diagonal anisotropic media, conductivities σ^D and σ^B, instead of σ^E and σ^H, are used. As a consequence, the GMIPML absorber has the property of material independence. This results in that Berenger's PML that originally developed for absorbing waves propagating in isotropic media can be simply and effectively extended to arbitrary anisotropic materials. Furthermore, due to the special feature (i.e., the material independence) of the proposed GMIPML absorber, it can also be used to absorb waves propagating in materials consisting of loss, dispersion, and non-linearity with slight modifications. Finally, it is worth mentioning that using only **D** and **B** fields is sufficient for the GMIPML because **E** and **H** can be replaced (see Eq. (1)) by **D** and **B** with their corresponding relations.

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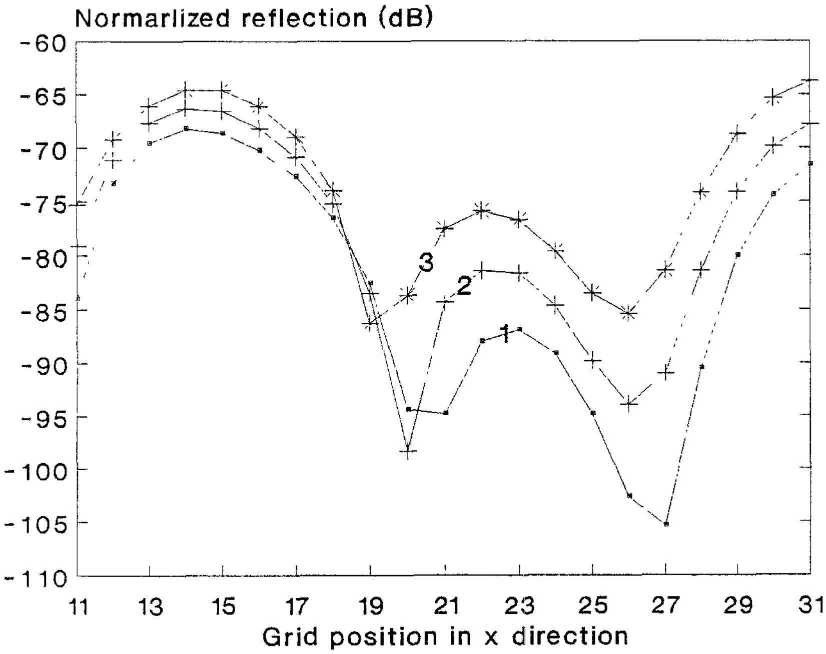

Fig. 1 Normalized local reflections for the E_z field component of the waves (with different values of θ) along the line (x, 11, 20.5); where curves 1 - 3 are for $\theta = \pi/6, \pi/4$, and $\pi/3$, respectively.